UT-962

Supersymmetric Grand Unification Model with the Orbifold Symmetry Breaking in the Six Dimensional Supergravity

T. Watari[a] and T. Yanagida[a,b]

[a] *Department of Physics, University of Tokyo, Tokyo 113-0033, Japan*

[b] *Research Center for the Early Universe, University of Tokyo, Tokyo 113-0033, Japan*

Abstract

We construct supersymmetric (SUSY) grand unification (GUT) models in the six dimensional space-time where the GUT symmetry is broken down to the standard-model gauge group by a simple orbifolding $\mathbf{T^2/Z_4}$ or $\mathbf{T^2/Z_6}$ and a pair of massless Higgs doublets in the SUSY standard model are naturally obtained. Since the background geometry here is simple compared with models using the Scherk-Schwarz mechanism, one might hope for an approximate gauge coupling unification in the present models. Here, the presence of the massless Higgs multiplets in the bulk is quite natural, since the anomaly cancellation in the six dimensional space-time requires N=2 hyper multiplets in the bulk, some of which are origins of the Higgs doublets. However, the origin of the quarks and leptons is still not clear at all.

Search for a solution to the doublet-triplet splitting problem in the supersymmetric (SUSY) grand unification theory (GUT) has led us to consider various extensions of the minimal SU(5) SUSY-GUT[1, 2, 3]. Recently, Kawamura[4] has pointed out an interesting solution to this problem utilizing an $\mathbf{S}^1/\mathbf{Z}_2$ orbifold in a five dimensional space-time, which is deeply related to the early suggestion by Witten[5][1]. Although this original model is non SUSY-GUT, it is easily extended to the SUSY-GUT if one assumes an $\mathbf{S}^1/(\mathbf{Z}_2\langle\sigma_1\rangle \times \mathbf{Z}_2'\langle\sigma_2\rangle)$ orbifold[7, 8]. Here, this orbifold is also regarded as $\mathbf{R}^1/(\mathbf{Z}\langle\sigma_1\sigma_2\rangle \times' \mathbf{Z}_2\langle\sigma_1\rangle)$, where the $\mathbf{Z}\langle\sigma_1\sigma_2\rangle$ gives the symmetry-breaking boundary condition a la Scherk-Schwarz[9]. A number of interesting features in this approach have been discussed[10]. However, it is claimed[11] recently that the Scherk-Schwarz breaking is equivalent to the Wilson line breaking[12] localized at a fixed point. Thus, the nontrivial background of gauge field exists at the fixed point, and its effect to the tree level gauge coupling is incalculable and it may be beyond the naive dimensional analysis[8][2]. Therefore, it is not necessarily obvious to maintain the gauge coupling unification even at the tree level.

The purpose of this letter is to show that a SUSY-GUT model with the orbifold GUT breaking mechanism is constructed in the six dimensional space-time without the Scherk-Schwarz breaking[3]. In this letter, we use the term "orbifold GUT breaking model" as a class of models that do not use the Scherk-Schwarz (or boundary breaking) mechanism. Since the background geometry here is simple compared with models using the Scherk-Schwarz mechanism, one might hope for an approximate gauge coupling unification in the present models. We consider a simple orbifold $\mathbf{T}^2/\mathbf{Z}_4$. We find that only a pair of the Higgs doublets $H_{\rm f}$ and $\bar{H}_{\rm f}$ survive together with the gauge multiplets of $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ in the extra two dimensional bulk. The matter multiplets $\mathbf{5}^*$ and $\mathbf{10}$ are assumed to reside on a fixed point that preserves SU(5) symmetry. We

[1] There has been proposed another kind of solution to the doublet-triplet splitting problem in a higher dimensional space-time[6].

[2] In the string theory the SU(5) gauge symmetry is realized on the branes where five D-branes coincide, and the Wilson line breaking corresponds to a spatially parallel separation of the five D-branes in a T-dual manifold[13]. The Wilson line breaking localized at fixed points implies that transverse fluctuation modes of D-branes (including their winding modes) develop expectation values differently on the three and two D-branes. Therefore, there is no reason that the expectation value of dilaton takes the same value at the "positions" of the separated SU(3) three branes and SU(2) two branes. Since the values of dilaton correspond to the gauge couplings, the coupling constants of each gauge groups are, in general, different from each others.

[3] There has been, recently, discussed the SO(10) GUT in the six dimensional space-time[14]. However, the Scherk-Schwarz boundary breaking is still postulated.

also find that a similar model can be constructed on another simple orbifold $\mathbf{T}^2/\mathbf{Z}_6$.

We first note that the five dimensional space-time is too small to have the desired orbifold GUT breaking model (without the Scherk-Schwarz mechanism). The orbifold projection associated with the $\mathbf{S}^1/\mathbf{Z}_2$ compactification is not enough to eliminate all the unwanted particles contained in the N=2 SUSY SU(5) multiplets. Such an elimination is possible only by using the Scherk-Schwarz mechanism in the $\mathbf{S}^1/(\mathbf{Z}_2\times\mathbf{Z}_2') \simeq \mathbf{R}^1/(\mathbf{Z}\times' \mathbf{Z}_2)$ compactification. Therefore, we consider the orbifold GUT breaking model in the six dimensional space-time. Furthermore, there is another reason to assume the higher dimensional space-time; the R-symmetry that is a crucial and unique symmetry to forbid a constant term in the superpotential[15] may arise from a rotation symmetry in the extra space. In this sense the present analysis provides the first basic and natural GUT model in a higher dimensional space-time[4]. We restrict ourselves to the six dimensional supergravity in this letter, since if we go further the consistency condition becomes more restrictive. We do not consider that the effective field theory should be necessarily formulated in the ten dimensional supergravity, even if the underlying fundamental theory is given by the string theory.

The possible rotational symmetry that the two dimensional torus $\mathbf{T}^2$ in the extra dimensional space possesses is $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$ or $\mathbf{Z}_6$. These are the symmetries that rotate the 4th-5th plane by π, $2\pi/3$, $\pi/2$ and $\pi/3$, respectively. We require that the orbifold projection condition explicitly distinguishes the color SU(3) and the flavor SU(2) of the SU(5) indices and breaks the SU(5) GUT down to the standard-model gauge group.

The N=2 SU(5) vector multiplet propagates in the six dimensional bulk. The lengths L of the extra dimensions are assumed to be of order of 1/(GUT scale) and the fundamental scale of the theory M_* is of order 10^{17}GeV. This lower cut-off scale is a crucial point in keeping the approximate gauge coupling unification at the effective GUT scale $\sim 10^{16}$GeV[8, 16]. The volume of the extra two dimensional space[5] is about $(M_* L)^2 \sim 10^2$.

Now that the SU(5) vector multiplet propagates in the six dimensional bulk, the six dimensional box anomaly must be canceled. Six dimensional box anomalies consist of

[4]The present models will serve as a useful framework for various investigations (including the confirmation of the gauge coupling unification) of orbifold GUT breaking models in a higher dimensional space-time.

[5]This large extra dimensions may be welcome to the gaugino mediation scenario of SUSY breaking[17].

pure gauge anomalies $\mathrm{tr}(F^4)$ and $(\mathrm{tr}(F^2))^2$, a gauge-gravity mixed anomaly $\mathrm{tr}(F^2)\mathrm{tr}(R^2)$ and pure gravitational anomalies[18]. Among these, pure gravitational anomalies can be canceled by introduction of SU(5) singlet fields and reducible anomalies $(\mathrm{tr}(F^2))^2$ and $\mathrm{tr}(F^2)\mathrm{tr}(R^2)$ can be canceled by the Green-Schwarz mechanism[19]. Thus we only care about the pure gauge $\mathrm{tr}(F^4)$ anomaly of the SU(5) gauge theory. In (1,0)-SUSY (N=2 SUSY in four dimensional sense) gauge theories in six dimensions, N=2 hyper multiplets have chirality opposite to that of the N=2 vector multiplet. Therefore, the pure gauge box anomaly from the gauge fermions of the N=2 SU(5) vector multiplet can be canceled by introducing N=2 hyper multiplets. The anomaly of the SU(n) N=2 vector multiplet is $-2n$ times that of the N=2 hyper multiplet in SU(n)-fundamental representation (up to reducible anomaly)[18], and hence we introduce ten ($\mathbf{5}$+$\mathbf{5}^*$) hyper multiplets in the six dimensional bulk[6].

We expect that the four dimensional N=1 vector multiplets of the $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ in the minimal SUSY standard model (MSSM) comes from the above N=2 SU(5) vector multiplet. Since we introduced N=2 hyper multiplets in $\mathbf{5}$+$\mathbf{5}^*$ representation, it is possible that the H_f and $\bar{H}_\mathrm{f}$ N=1 chiral multiplets in the MSSM also originate from the bulk fields. Spectrum of massless particles that live in the bulk are determined by the orbifold projection condition that distinguishes the color SU(3) and the flavor SU(2), and hence the remaining particles may be chosen as only doublets by adopting an appropriate orbifold projection as shown below. The phenomenological reason for which we identify the Higgs as bulk fields (not as fixed point fields), will be explained later. On the contrary, we postulate by hand that the quarks and leptons N=1 chiral multiplets, $(\mathbf{5}^* + \mathbf{10})$, reside on orbifold fixed points. This is the weakest point in the present approach.

In the model construction of the orbifold GUT breaking, the existence of the fixed point that preserves the SU(5) symmetry is an important ingredient[8]. A natural explanation of the anomaly cancellation in terms of the SU(5) GUT which is nothing but a miracle in the standard model, the charge quantization of the $\mathrm{U}(1)_Y$, and the bottom-tau Yukawa unification are the major reasons that we believe SUSY-GUT along

[6]The anomaly from the N=2 vector multiplet is completely canceled by the N=2 hyper multiplet in the adjoint representation. Here, they may form a vector multiplet of the N=4 SUSY. Additional three $(\mathbf{5} + \mathbf{5}^*)$ hyper multiplets and a $(\mathbf{10} + \mathbf{10}^*)$ hyper multiplet do not give rise to an irreducible pure gauge anomaly, and these multiplets may be of phenomenological use. In this letter, however, we only discuss the simplest possibility (ten $\mathbf{5} + \mathbf{5}^*$ in the text).

with the gauge coupling unification suggested from the experiments. The above three features are still maintained if the orbifold geometry has a fixed point that preserves the four dimensional SU(5) GUT symmetry even though the orbifold GUT breaking model has no complete higher dimensional SU(5) symmetry. We consider the model in which the three families of quarks and leptons $\mathbf{5}^* + \mathbf{10}$ reside on such a fixed point.

In order to have a fixed point which preserves the SU(5) symmetry, the orbifold group must have nontrivial and proper subgroup. First, the isotropy group associated to such a fixed point[7] is not trivial by definition. Secondly, if the isotropy group were identical to the whole orbifold group, then the orbifold projection associated to the isotropy group of such a fixed point would distinguish the color SU(3) and the flavor SU(2) subgroups of the SU(5), and hence the SU(5) symmetry would not be preserved. Therefore, the isotropy group of such a fixed point is nontrivial and proper subgroup of the whole orbifold group. This means that the orbifold group candidate are $\mathbf{Z_4}$ and $\mathbf{Z_6}$ since $\mathbf{Z}_2$ and $\mathbf{Z}_3$ do not have such a subgroup. Therefore, we consider the $\mathbf{T}^2/\mathbf{Z}_4\left<\sigma\right>$ and $\mathbf{T}^2/\mathbf{Z}_6\left<\sigma\right>$ model where the σ is the generator of the each orbifold group $\mathbf{Z}_4$ and $\mathbf{Z}_6$.

Let us first consider the model on $\mathbf{T}^2/\mathbf{Z}_4\left<\sigma\right>$ orbifold. The generator σ of the $\mathbf{Z}_4$ transformation rotates the 4th-5th plane by $\pi/2$:

$$(z \equiv (x_4 + ix_5)) \rightarrow \sigma \cdot z = e^{i(\theta = \frac{2\pi}{4})} z. \tag{1}$$

The geometric picture of the $\mathbf{T}^2/\mathbf{Z}_4\left<\sigma\right>$ orbifold is shown in Fig.1. There are two $\mathbf{Z}_4\left<\sigma\right>$ fixed points and there is one $\mathbf{Z}_2\left<\sigma^2\right>$ fixed point[8]. We can identify the latter fixed point as the SU(5) preserving fixed point, as seen below. The N=2 bulk fields consist of an N=1 SU(5) vector multiplet $V(z)$, an N=1 chiral multiplet $\Sigma(z)$ that transforms as an adjoint representation of the SU(5), and N=1 chiral multiplets $F_j(z)$ and $\bar{F}^j(z)$ (j=1,...,10) that transforms $\mathbf{5}$ and $\mathbf{5}^*$ before the orbifold projection. The massless spectrum of the gauge theory on the orbifold is given by the zero modes that satisfy the following orbifold projection condition:

$$V(x,z) = \widetilde{\gamma}_\sigma V(x, e^{i\theta} z)\widetilde{\gamma}_\sigma^{-1}, \quad \Sigma(x,z) = e^{i\theta}\widetilde{\gamma}_\sigma \Sigma(x, e^{i\theta} z)\widetilde{\gamma}_\sigma^{-1}, \tag{2}$$

[7]Isotropy (sub)group of a point is a subgroup of a transformation group, say the orbifold group, which consists of elements that fix the point.

[8]In this letter, we call the fixed point whose isotropy group is $\mathbf{Z}_2\left<\sigma^2\right>$ as $\mathbf{Z}_2\left<\sigma^2\right>$ fixed point for brevity. Similar terminology is also used later in the $\mathbf{T}^2/\mathbf{Z}_6$ orbifold model.

$$F(x,z)_j = e^{i\theta n_j}\widetilde{\gamma}_\sigma F(x, e^{i\theta}z)_j, \quad \bar{F}(x,z)^j = e^{i\theta(-1-n_j)}\widetilde{\gamma}_\sigma^{-1}\bar{F}(x, e^{i\theta}z)^j, \tag{3}$$

where $\theta = (2\pi)/4$. Rotational charges n_j for the hyper multiplet can be $n/2$ ($n = 0, 1, 2, 3, ...$). $\widetilde{\gamma}_\sigma$ is the (5×5) gauge twisting matrix associated to the generator σ that must satisfy $(\widetilde{\gamma}_\sigma)^4 = \mathbf{1}$.

We take the gauge twisting matrix $\widetilde{\gamma}_\sigma$ as

$$\widetilde{\gamma}_\sigma = \mathrm{diag}(e^{i\theta m}, e^{i\theta m}, e^{i\theta m}, e^{i(\theta m+\pi)}, e^{i(\theta m+\pi)}), \tag{4}$$

where m is an arbitrary integer and $\theta = (2\pi)/4$. Under this choice, the $\mathbf{Z}_2$ $\langle\sigma^2\rangle$ fixed point preserves the SU(5) symmetry because the gauge twisting matrix $\widetilde{\gamma}_{\sigma^2} \equiv (\widetilde{\gamma}_\sigma)^2 \propto \mathbf{1}$ does not make any discrimination between the color SU(3) and the flavor SU(2). We put the three families of quarks and leptons on this fixed point.

Now we can see that the massless particles (Kaluza-Klein zero modes) from the N=2 SU(5) vector multiplet are just the N=1 vector multiplets of the MSSM. Only one pair of the N=1 chiral multiplets H_f from the F_1 and the $\bar{H}_\mathrm{f}$ from the $\bar{F}^2$ survive the orbifold projection conditions Eq.(3) if we take $n_1 = (2-m)$, $n_2 = (1-m)$ and n_j (j = 3,...,10) to be half integers. These are exactly the pair of Higgs doublets in the MSSM. No other unwanted particle remains massless. We can also see that the triangle anomalies which might appear at fixed points because of the orbifolding completely vanish[20].

We obtain the desired massless Higgs multiplets H_f and $\bar{H}_\mathrm{f}$ in the bulk. As a matter of fact, this is a necessary property as long as the third family of the quarks and leptons reside on the SU(5) preserving fixed point. The reason is the following. Suppose that the two Higgs doublets reside on one of the two $\mathbf{Z}_4$ $\langle\sigma\rangle$ fixed points and the third family reside on the $\mathbf{Z}_2$ $\langle\sigma^2\rangle$ fixed point. Let us consider how the Yukawa couplings in the superpotential are generated. Since the quark and lepton multiplets are separated from the two Higgs doublets by the distance $M_*L \sim 10$, an exchange of particles of mass of order of the fundamental scale M_* is not enough to induce the Yukawa couplings because of the damping of the wave function $e^{-M*L} \lesssim 10^{-4}$. Only the Kaluza-Klein particles of the $\mathbf{5}+\mathbf{5}^*$ hyper multiplets can do the job. However, we can see that all those Kaluza-Klein particles have zero wave function at the $\mathbf{Z}_4$ fixed points in models where no massless Higgs multiplet remains in the bulk. Therefore, necessary Yukawa couplings are not generated by the exchanges of the Kaluza-Klein particles.

It is easy to see that a similar argument to the above also holds in the model of

$\mathbf{T}^2/\mathbf{Z}_6 \langle\sigma\rangle$ orbifold. The generator σ rotates the 4th-5th plane by $\pi/3$:

$$z \to e^{i\theta} z \qquad \left(\theta = \frac{2\pi}{6}\right). \tag{5}$$

Orbifold projection conditions are the same as Eq.(2) and Eq.(3) with $\theta = (2\pi)/4$ replaced by $\theta = (2\pi)/6$. The (5×5) gauge twisting matrix associated to the generator σ can be given by

$$\widetilde{\gamma}_\sigma = \mathrm{diag}(e^{i\theta m}, e^{i\theta m}, e^{i\theta m}, e^{i(\theta m+\pi)}, e^{i(\theta m+\pi)}), \tag{6}$$

as in Eq.(4), or by

$$\mathrm{diag}(e^{i\theta m}, e^{i\theta m}, e^{i\theta m}, e^{i(\theta m\pm 2\pi/3)}, e^{i(\theta m\pm 2\pi/3)}), \tag{7}$$

where m is again an arbitrary integer. In the former case $\mathbf{Z}_3 \langle\sigma^2\rangle$ fixed point are the SU(5) preserving fixed point and in the latter case the $\mathbf{Z}_2 \langle\sigma^3\rangle$ fixed point preserves the SU(5) (see Fig.2). Precisely the N=1 vector multiplets of the MSSM survive the orbifold projection, and the two massless Higgs doublets remain in the bulk if we take the rotational charges n_j of the N=2 hyper multiplets as $n_1 = (-m-3)$, $n_2 = (-m+2)$ and $n_j = (-m-2), (-m-5)$, (half integers) for (j=3,...,10) in the former case, and $n_1 = (-m\mp 2)$, $n_2 = (-m\mp 2-1)$ and $n_j = (-m\mp 1-4)$, $(-m\mp 1-3)$, (half integers) for (j=3,...,10) in the latter case. We also see that the two Higgs doublets should be in the bulk to have the sufficiently large Yukawa couplings in this $\mathbf{T}^2/\mathbf{Z}_6$ orbifold model.

If we assume that the second and/or the first family also reside on the SU(5) preserving fixed point, then we have to find some mechanism to break the SU(5) relation $m_s = m_\mu$ and/or $m_d = m_e$. This may be realized through the mixing of these quarks and leptons with the massive multiplets that propagate in the bulk. In the $\mathbf{T}^2/\mathbf{Z}_6$ orbifold model such massive multiplets may be supplied by Kaluza-Klein towers of the hyper multiplets F_j and $\bar{F}^j$, and in the $\mathbf{T}^2/\mathbf{Z}_4$ orbifold model they are contained in the heavy particles at the cut-off scale M_*. A detailed phenomenological aspects of the present models will be given elsewhere.

Acknowledgments

We are grateful to Dr.Yasunori Nomura for useful discussion. T.W. thanks the Japan Society for the Promotion of Science for financial support. This work was partially supported by "Priority Area: Supersymmetry and Unified Theory of Elementary Particles (# 707)" (T.Y.).

References

[1] A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B **115** (1982) 380;
I. Antoniadis, J. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B **194** (1987) 231.

[2] S. Dimopoulos and F. Wilczek, NSF-ITP-82-07;
M. Srednicki, Nucl. Phys. B **202** (1982) 327.

[3] T. Yanagida, Phys. Lett. B **344** (1995) 211 [hep-ph/9409329];
J. Hisano and T. Yanagida, Mod. Phys. Lett. A **10** (1995) 3097 [hep-ph/9510277];
T. Hotta, K. I. Izawa and T. Yanagida, Phys. Rev. D **53** (1996) 3913 [hep-ph/9509201], Phys. Rev. D **54** (1996) 6970 [hep-ph/9602439];
K. I. Izawa and T. Yanagida, Prog. Theor. Phys. **97** (1997) 913 [hep-ph/9703350].

[4] Y. Kawamura, Prog. Theor. Phys. **103** (2000) 613 [hep-ph/9902423].

[5] P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B **258** (1985) 46;
E. Witten, Nucl. Phys. B **258** (1985) 75.

[6] Y. Imamura, T. Watari and T. Yanagida, hep-ph/0103251;
T. Watari and T. Yanagida, hep-ph/0108057;
C. Csaki, G. D. Kribs and J. Terning, hep-ph/0107266.

[7] Y. Kawamura, Prog. Theor. Phys. **105** (2001) 999 [hep-ph/0012125].

[8] L. Hall and Y. Nomura, Phys. Rev. D **64** (2001) 055003 [hep-ph/0103125].

[9] J. Scherk and J. H. Schwarz, Nucl. Phys. B **153** (1979) 61;
J. Scherk and J. H. Schwarz, Phys. Lett. B **82** (1979) 60.

[10] G. Altarelli and F. Feruglio, Phys. Lett. B **511** (2001) 257 [hep-ph/0102301];
T. Kawamoto and Y. Kawamura, hep-ph/0106163;
A. Hebecker and J. March-Russell, hep-ph/0106166;
N. Haba, T. Kondo, Y. Shimizu, T. Suzuki and K. Ukai, hep-ph/0108003.

[11] A. Hebecker and J. March-Russell, hep-ph/0107039.
See also, L. J. Hall, H. Murayama and Y. Nomura, hep-th/0107245.

[12] Y. Hosotani, Phys. Lett. B **126** (1983) 309, Annals Phys. **190** (1989) 233.

[13] J. Polchinski, *String Theory*, (Cambridge University Press,UK,1998).

[14] T. Asaka, W. Buchmuller and L. Covi, hep-ph/0108021;
L. Hall, Y. Nomura, T. Okui and D. Smith, hep-ph/0108071.

[15] K. I. Izawa and T. Yanagida, Prog. Theor. Phys. **97** (1997) 913 [hep-ph/9703350].

[16] R. Contino, L. Pilo, R. Rattazzi and E. Trincherini, hep-ph/0108102.

[17] D. E. Kaplan, G. D. Kribs and M. Schmaltz, Phys. Rev. D **62** (2000) 035010 [hep-ph/9911293];
Z. Chacko, M. A. Luty, A. E. Nelson and E. Ponton, JHEP **0001** (2000) 003 [hep-ph/9911323].
See also for the early work, K. Inoue, M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. D **45** (1992) 328.

[18] See, for example, N. Borghini, Y. Gouverneur and M. H. Tytgat, hep-ph/0108094, and references therein.

[19] M. B. Green and J. H. Schwarz, Phys. Lett. B **149** (1984) 117.

[20] N. Arkani-Hamed, A. G. Cohen and H. Georgi, hep-th/0103135.

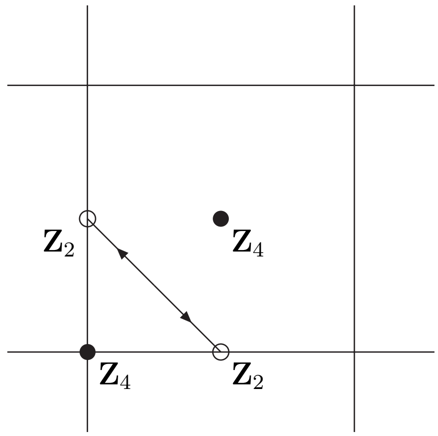

Figure 1: $\mathbf{T}^2/\mathbf{Z}_4$ orbifold geometry is described. There are two $\mathbf{Z}_4\,\langle\sigma\rangle$ fixed points ($\bullet$'s in the figure) and one fixed point ($\circ$ in the figure) whose isotropy group is $\mathbf{Z}_2\,\langle\sigma^2\rangle$. The arrow denotes the identification between mirror images under $\mathbf{Z}_4/\mathbf{Z}_2$. This $\mathbf{Z}_2\,\langle\sigma^2\rangle$ fixed point is the SU(5) preserving fixed point.

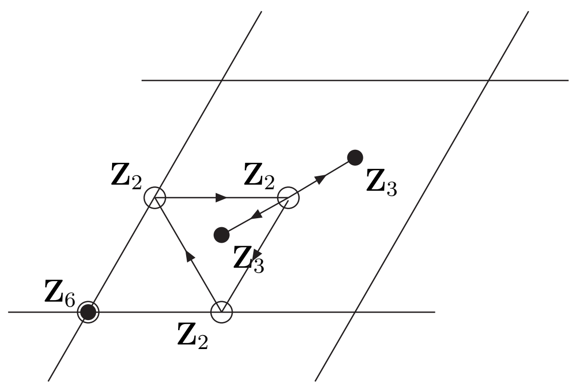

Figure 2: Geometry of $\mathbf{T}^2/\mathbf{Z}_6$ is described. This orbifold has three fixed points whose isotropy groups are all different: namely the $\bullet(\mathbf{Z}_3\,\langle\sigma^2\rangle$ fixed), the $\circ(\mathbf{Z}_2\,\langle\sigma^3\rangle$ fixed) and the $\bullet$-$\circ(\mathbf{Z}_6\,\langle\sigma\rangle$ fixed) in this figure. Each of the fixed point $\bullet$ and $\circ$ can be an SU(5) preserving fixed point. Arrows denote the identification between mirror images under $\mathbf{Z}_6/\mathbf{Z}_3$ ($\bullet$'s) and $\mathbf{Z}_6/\mathbf{Z}_2$ ($\circ$'s), respectively.